

Closed-Loop Supply Chain Model with Government Participation

Yunfu Huo^{1,a} and Xinxin Chen^{2,b,*}

¹ Economic and management Dalian university No.10, Xuefu Ave, Economic Technological Development Zone, Dalian 116622, P. R. China

² Economic and management Dalian university No.10, Xuefu Ave, Economic Technological Development Zone, Dalian 116622, P. R. China

*²Corresponding author: 1179142921@qq.com

Keywords: Game theory, Willingness to pay; Recycled quality, Closed-loop supply chain, The government subsidies

Abstract. In this paper, game theory is used to construct a closed-loop supply chain with a single manufacturer and consumer, and the decision-making model of the closed-loop supply chain under government subsidies is established, the optimal decision of the model is obtained, and the results are discussed. The research shows that improving the consumer preference coefficient helps to increase the price of remanufactured products and the profits of manufacturers and retailers. Under the condition that the consumer preference coefficient is determined, strengthening the supervision of the recycling stage can effectively improve the sales of new products without affecting the price of new products. When the consumer preference coefficient is in different range, the change of waste product quality coefficient will have different influence on the consumer's profit. Finally, the model is verified by numerical examples and simulation analysis.

1 The introduction

With the continuous innovation of science and technology in contemporary society, the replacement rate of new products is also accelerated, so that there are more and more waste products, which leads to more and more serious resource and environmental problems. If these waste products can not be properly disposed, it will cause a lot of waste of resources and environmental pollution. Therefore, in order to better promote the recycling of waste products, the State Council issued several opinions on accelerating the development of circular economy in 2005 to support the development of remanufacturing. In 2009, China promulgated the measures for replacing old appliances with new ones. Most waste household appliances contain a large number of recyclable materials, and the recycling of waste products can greatly reduce the pollution to the environment. Therefore, more and more attention has been paid to the recycling and remanufacturing of household appliances, and enterprises and governments have begun to pay attention to this issue.

In recent years, more and more people pay attention to the recycling of resources. Therefore, more and more scholars begin to pay attention to the issues related to closed-loop supply chain. Weidong Wang et al. ^[4] studied the model by establishing the decision-making model of manufacturers and retailers in competition and information asymmetry and introducing the joint contract composed of wholesale price and cost sharing. Ying Chen et al. ^[5] set up a newsboy model based on the supply chain composed of manufacturers and retailers, and optimized the supply chain by using wholesale price and buyback contracts. Jian Cao et al. ^[6] established and discussed three ways of remanufacturing product cost, namely manufacturer recycling, retailer recycling and recycler recycling, and analyzed it from the perspectives of enterprise income, consumer surplus and recovery rate. Savaskan ^[7] studied the impact of different recycling modes on pricing strategies of different node members of the supply chain. In the case of government participation, most scholars currently assume that the government undertakes a single subsidy. Zhiqiang Peng et al. ^[1] compared and analyzed the closed-loop supply chain model under differential pricing by separately subsidizing manufacturers and retailers. Meng Li ^[2] adopted two kinds of subsidies for manufacturers and consumers, and found that when the government adopted two different subsidies, the sales of new products would be reduced and the demand for remanufactured products would be increased. Wenwei Gong et al. ^[3] joined a third-party waste recycler and compared and analyzed the decision changes of each member of the closed-loop supply chain with or without government subsidies. However, there are still relatively few literatures considering the two subsidy modes at the same time. Faxin Cheng et al. ^[8] brought consumers' green preference into consideration and proposed relevant decisions by simultaneously subsidizing manufacturers and consumers and establishing steinberg model. Tong Shu et al. ^[9] established a remanufacturing game model by taking into account the differences in consumers' willingness to pay and the different allocation proportions of government subsidies between consumers and manufacturers.

To sum up, the current study of closed-loop supply chain involving the government involvement in the literature, mostly is the study of single government subsidies, few research papers will government subsidy according to certain proportion for both subsidies, yet at the same time consider difference consumer willingness to pay and recycling of uncertain quality related literature is less. Therefore, this paper will take into account consumers' different willingness to pay and the uncertain quality of recovery, and study the impact of subsidy on closed-loop supply chain decision-making

for manufacturers and consumers at the same time, so as to find the optimal subsidy proportion, and verify the effectiveness of the model by combining with example analysis.

2 Model establishment.

2.1 Model description and basic assumptions.

This paper sets up a closed-loop supply chain system composed of manufacturers and consumers with government participation. In this system, the manufacturer is responsible for recycling waste products from consumers and renovating waste products to produce remanufactured products, while the manufacturer also USES raw materials to produce new products. After production is completed, the manufacturer sells the new product and the remanufactured product to consumers. In order to encourage enterprises to remanufacture and promote the recycling of resources, the government will choose to subsidize both consumers and manufacturers.

In order to solve the research problem in this paper, the following basic assumptions should be made:

(1) members of the closed-loop supply chain form a game between manufacturers and consumers. The manufacturer is the leader of the game, and the consumer is the follower. The members' information about the quality of waste products is asymmetric, and their decision-making preference is set as risk neutral, and they all make decisions according to the maximization of their own interests.

(2) assuming that the market size is normalized to 1, the new product produced by the manufacturer and the remanufactured product are heterogeneous.

(3) The government subsidizes both manufacturers and consumers. Government departments subsidize remanufactured products, and $\frac{C_r}{C_n} X$ subsidies to manufacturers, $(1 - \frac{C_r}{C_n}) X$ subsidies to consumers.

2.2 Symbol description.

Based on the research of shu tong et al. [9], this paper adopts different subsidy proportions and takes into account the uncertainty of the recycling quality of waste products. The setting of some symbols is consistent with the above literatures.

Table 1 Description of symbols and descriptions

Symbol	Describe	Symbol	Describe
q	True recovery quality	λ	The quality factor that consumers lie about
F ($F = \theta\lambda q$)	Manufacturer's buyback price	θ	Quality value coefficient
G ($G = h + kF$)	The recycling number	h	Minimum market recovery
k	Recovery price sensitivity factor	C_n 、 C_r ($C_r = C_n - v\lambda q$)	The manufacturing cost of a new, remanufactured product
v	Quality cost factor	α_n	Willingness to pay for new products (subject to uniform distribution on [0,1])
δ ($0 < \delta < 1$)	Consumer preference coefficient	$f(\alpha_n)$ 、 $F(\alpha_n)$	Density, distribution function
p_n 、 p_r	The selling price of a new, remanufactured product	q_n 、 q_r	Sales of new and remanufactured products
X ($C_n > X$)	The amount of subsidy per unit remanufactured product	U_n 、 U_r	The residual utility of a consumer buying a new, remanufactured product
Π_m 、 Π_c	Profits of manufacturers and consumers		

2.3 Model establishment.

2.3.1 Analysis of consumer demand.

Under the strategy of government subsidies, it is obtained from the research of Tong Shu et al. ^[9] that when the utility brought by purchasing new products and remanufacturing products is the same,

$\alpha_n = \frac{p_n - p_r + (1 - \frac{C_r}{C_n})X}{1 - \delta}$. Therefore, the residual utility of consumers buying new products and remanufacturing products is:

$$U_n = \int_{\frac{p_n - p_r + (1 - \frac{C_r}{C_n})X}{1 - \delta}}^1 (\alpha_n - p_n) d\alpha_n. \quad (1)$$

$$U_r = \int_{\frac{p_r - (1 - \frac{C_r}{C_n})X}{\delta}}^{\frac{p_n - p_r + (1 - \frac{C_r}{C_n})X}{1 - \delta}} \left[\delta \alpha_n - p_r + (1 - \frac{C_r}{C_n})X \right] d\alpha_n. \quad (2)$$

The market demand for new products and remanufactured products is:

$$q_n = 1 - \frac{p_n - p_r + (1 - \frac{C_r}{C_n})X}{1 - \delta}. \quad (3)$$

$$q_r = \frac{\delta p_n - p_r + (1 - \frac{C_r}{C_n})X}{(1 - \delta)\delta}. \quad (4)$$

2.3.2 Analysis of manufacturer's optimal decision.

The profit function of the manufacturer is:

$$\Pi_m = (p_n - C_n)q_n + (p_r - C_r + \frac{C_r}{C_n}X)q_r - FG. \quad (5)$$

By substituting Eq. 3 and Eq. 4 into Eq. 5, the following equation can be obtained:

$$\begin{aligned} \max \Pi_m &= (p_n - C_n)q_n + (p_r - C_r + \frac{C_r}{C_n}X)q_r \\ &= (p_n - C_n) \left[1 - \frac{p_n - p_r + (1 - \frac{C_r}{C_n})X}{1 - \delta} \right] + (p_r - C_r + \frac{C_r}{C_n}X) \frac{\delta p_n - p_r + (1 - \frac{C_r}{C_n})X}{(1 - \delta)\delta} - FG. \end{aligned} \quad (6)$$

Since the manufacturer is strictly concave with respect to p_n, p_r , the model has a unique solution. Therefore, Π_m calculates the first derivative of p_n, p_r and sets it to zero, and then:

$$p_n^* = \frac{1 + C_n}{2}. \quad (7)$$

$$p_r^* = \frac{X + C_r + \delta - 2\frac{C_r}{C_n}X}{2}. \quad (8)$$

By substituting Eq. 7 and Eq. 8 into Eq. 3 and Eq. 4 respectively, the output of the new product and the remanufactured product can be obtained as follows:

$$q_n^* = \frac{C_r - C_n - X}{2(1 - \delta)} + \frac{1}{2}. \quad (9)$$

$$q_r^* = \frac{\delta C_n - C_r + X}{2\delta(1 - \delta)}. \quad (10)$$

By substituting Eq. 7, Eq. 8, Eq. 9 and Eq. 10 into equations Eq. 6, Eq. 3 and Eq. 4, it can be concluded that the optimal profit function of new products and remanufactured products of manufacturers and consumers is:

$$\Pi_m^* = \frac{(1-C_n)(1-\delta-X-\nu\lambda q)}{4(1-\delta)} + \frac{(X+\delta-C_n+\nu\lambda q)(\delta C_n-C_n+X+\nu\lambda q)}{4\delta(1-\delta)} - h\theta\lambda q - k\theta^2\lambda^2q^2. \quad (11)$$

$$U_n^* = \left[\frac{X+\nu\lambda q}{2(1-\delta)} + \frac{1}{2} \right] \left[\frac{2C_n-2\delta C_n-X-\nu\lambda q}{4(1-\delta)} + \frac{1}{4} \right] - \frac{C_n}{2}. \quad (12)$$

$$U_r^* = \frac{(\delta C_n+X-C_n+\nu\lambda q)^2}{8\delta(1-\delta)^2}. \quad (13)$$

The revenue function of consumers is:

$$\begin{aligned} \Pi_c^* &= U_n^* + U_r^* \\ &= \left[\frac{X+\nu\lambda q}{2(1-\delta)} + \frac{1}{2} \right] \left[\frac{2C_n-2\delta C_n-X-\nu\lambda q}{4(1-\delta)} + \frac{1}{4} \right] + \frac{(\delta C_n+X-C_n+\nu\lambda q)^2}{8\delta(1-\delta)^2} + h\theta\lambda q + k\theta^2\lambda^2q^2 - \frac{C_n}{2} \end{aligned} \quad (14)$$

2.4 Model analysis.

In order to ensure that the proposed solution is meaningful, the corresponding conditions are satisfied for the following parameters: $0 < k, 0 < \nu < 1, 0 < \delta < 1, 0 < X < C_r < C_n < 1$.

Conclusion 1. Under the condition that the quality coefficient of waste products reported by consumers remains unchanged, the price of remanufactured products, the profit of manufacturers and the profit of consumers will increase with the increase of consumer preference coefficient, while the sales volume of new products will decrease with the increase of consumer preference coefficient.

$$\text{Proof: } \frac{\partial p_r}{\partial \delta} = \frac{1}{2} > 0, \quad \frac{\partial q_n}{\partial \delta} = -\frac{(\nu\lambda q + X)}{2(1-\delta)^2} < 0,$$

$$\begin{aligned} \frac{\partial \pi_m}{\partial \delta} &= \frac{(C_n-1)(X+\nu\lambda q)}{4(1-\delta)^2} + \frac{\delta^2[(1+C_n)(X-C_n+\nu\lambda q)] - (1-2\delta)(X-C_n+\nu\lambda q)^2}{4\delta^2(1-\delta)^2} \\ &> \frac{(C_n-1)(X+\nu\lambda q)}{4(1-\delta)^2} + \frac{(X-C_n+\nu\lambda q)(1+2C_n-X)+C_n}{4(1-\delta)^2} > 0, \\ \frac{\partial \pi_c}{\partial \delta} &= \frac{(\delta C_n+X-C_n+\nu\lambda q)[C_n(\delta^2-2\delta+1)+(\nu\lambda q+X)(3\delta-2\delta^2-1)]}{8\delta^2(1-\delta)^3} \\ &> \frac{(\delta C_n+X-C_n+\nu\lambda q)(\nu\lambda q+X)}{8\delta(1-\delta)^2} > 0 \end{aligned}$$

Conclusion 1 shows that sales of new products and consumer preference coefficient negative correlation, because consumer preference coefficient represents the remanufacturing product quality, so improving the quality of the remanufacturing product will affect the sales of new products, this requires manufacturers in the production of remanufacturing products, control of remanufacturing product quality; When the quality coefficient of waste products reported by consumers is constant, the improvement of consumer preference coefficient is conducive to the improvement of profits of consumers and manufacturers.

Conclusion 2. When the preference coefficient of consumers remains unchanged, the quality coefficient of waste products reported by consumers will increase, the sales price of remanufactured products will increase, the demand for new products will decrease, and the demand for remanufactured products will increase.

$$\text{Proof: } \frac{\partial p_r}{\partial \lambda} = \frac{\nu q}{2} \left(\frac{2}{C_n} - 1 \right) > 0, \quad \frac{\partial q_n}{\partial \lambda} = \frac{-\nu q}{2(1-\delta)} < 0, \quad \frac{\partial q_r}{\partial \lambda} = \frac{\nu q}{2(1-\delta)} > 0$$

Conclusion 2 shows that when the consumer preference coefficient remains unchanged, the price of remanufactured products, the sales volume of remanufactured products are positively correlated with the quality coefficient of waste products reported by consumers, while the sales volume of new products is negatively correlated with the quality coefficient of waste products reported by consumers. When consumers improve the quality of reported waste products, the sales of new products will decrease, which requires manufacturers to strengthen the supervision of waste products when recycling waste products..

Conclusion 3. No matter how the preference coefficient of consumers and the quality coefficient of waste products reported by consumers change, the profit of consumers is always the minimum value.

Proof: When $\frac{\partial^2 \pi_c}{\partial \lambda^2} = \frac{v^2 q^2}{4\delta(1-\delta)} + 2k\theta^2 q^2 > 0$ so 1 opens up;

Assuming that $\frac{\partial \pi_c}{\partial \lambda} = 0$ so $\lambda_c = \frac{v(\delta C_n - C_n + X) + 4\delta(1-\delta)h\theta}{[8\delta(\delta-1)k\theta^2 - v^2]q}$ there is a minimum of

consumer profit.

Conclusion 3 shows that when the consumer preference coefficient and the quality coefficient of waste products reported by consumers are any value, the total profit of consumers will reach the minimum value. In this case, the manufacturer's preference coefficient will affect the quality of waste products reported by consumers in the recycling process.

Conclusion 4. When the manufacturer determines $0 < \delta < \delta_1$ or $\delta_2 < \delta < 1$, the manufacturer and the consumer obtain the minimum profit at λ_m or λ_c respectively. When the quality factor of waste products increases or decreases, the profit of manufacturers and consumers will increase.

Proof: Assuming that $\frac{\partial^2 \pi_m}{\partial \lambda^2} = \frac{[v - 4\delta(1-\delta)k\theta^2 q]q}{2\delta(1-\delta)} = 0$,

$$\delta_1 = \frac{1}{2} - \frac{\sqrt{(kq\theta)^2 - kqv}}{2kq\theta}, \quad \delta_2 = \frac{1}{2} + \frac{\sqrt{(kq\theta)^2 - kqv}}{2kq\theta}$$

When $0 < \delta < \delta_1$ or $\delta_2 < \delta < 1$, $\frac{\partial^2 \pi_m}{\partial \lambda^2} > 0$, the opening of π_m is upward, and there is a minimum;

Assuming that $\frac{\partial \pi_m}{\partial \lambda} = 0$ $\lambda_m = \frac{v(\delta C_n - C_n + X) - 2\delta(1-\delta)h\theta}{[4\delta(1-\delta)k\theta^2 - v^2]q}$

Conclusion 4 shows that when the consumer preference coefficient is $0 < \delta < \delta_1$ or $\delta_2 < \delta < 1$, the quality coefficient of waste products increases or decreases, which will increase the profit of both manufacturers and consumers. At this time, the government should strengthen the supervision of the recycling link to avoid the phenomenon of collusion between manufacturers and consumers.

Conclusion 5. In the case of $\delta_1 < \delta < \delta_2$, when the quality coefficient of waste products reported by consumers is equal to the quality coefficient of waste products expected by manufacturers, the consumer preference coefficient is

$\delta = \frac{C_n + \frac{hv}{2k\theta} - X}{C_n}$. At this time, the profit of the manufacturer and the consumer reaches the maximum value and

minimum value respectively in $\delta = \frac{C_n + \frac{hv}{2k\theta} - X}{C_n}$ places, and the subsidy ratio of the government to the

manufacturer is $\frac{C_r}{C_n} = 1 - \frac{1}{2k\theta C_n} + \frac{(h-1)v^2 C_n^2}{2k\theta C_n \left[4k\theta^2 (C_n + \frac{hv}{2k\theta} - X)(X - \frac{hv}{2k\theta}) - v^2 C_n^2 \right]}$.

Proof: Assuming that $\lambda_m = \lambda_c$, $\delta = \frac{C_n + \frac{hv}{2k\theta} - X}{C_n}$;

$$\frac{\partial \lambda_m}{\partial \delta} = \frac{4(1-\delta)^2 k \nu \theta^2 C_n - \nu^3 C_n + 2(1-2\delta) h \nu^2 \theta - 4(1-2\delta) k \nu \theta^2 X}{[4\delta(1-\delta)k\theta^2 - \nu^2]^2 q},$$

$$\frac{\partial \lambda_c}{\partial \delta} = \frac{-2 \frac{\partial \lambda_m}{\partial \delta} + \nu^3 C_n}{[8\delta(\delta-1)k\theta^2 - \nu^2]^2 q},$$

$$\frac{C_r}{C_n} = 1 - \frac{1}{2k\theta C_n} + \frac{(h-1)\nu^2 C_n^2}{2k\theta C_n \left[4k\theta^2 (C_n + \frac{h\nu}{2k\theta} - X)(X - \frac{h\nu}{2k\theta}) - \nu^2 C_n^2 \right]}$$

Conclusion 5 shows that when the consumer preference coefficient is $\delta_1 < \delta < \delta_2$, the manufacturer's profit function is convex with respect to consumer preference coefficient, and the consumer's profit function is concave with respect to consumer preference coefficient. Increasing or decreasing the consumer preference coefficient is beneficial to increase the manufacturer's profit, but it will decrease the consumer's profit. When the manufacturer's profit is optimal, the consumer's profit is the lowest, and the subsidy proportion distribution is not optimal.

3 Example analysis.

In order to verify the correctness of the above conclusion, the conclusion is further illustrated by numerical examples. The basic parameter is set to $C_n = 0.6, X = 0.2, \theta = 0.1, q = 0.2, h = 0.3, k = 0.2, \nu = 0.2$.

Fig. 1 also should be presented as part of the text. Color figures are welcome for the online version of the journal. Generally, these figures will be reduced to black and white for the print version. The author should indicate on the checklist if he wishes to have them printed in full color and make the necessary payments in advance.

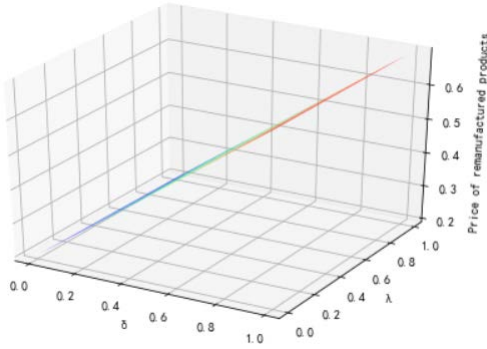


Figure 1. Influence of λ , δ on p_r

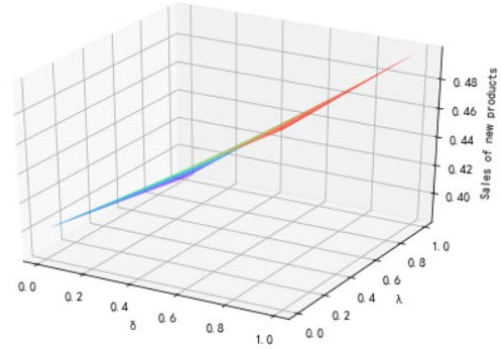


Figure 2. Influence of λ , δ on q_n

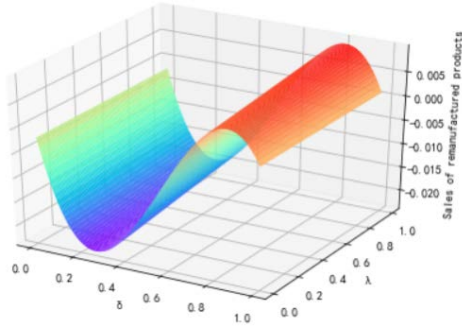


Figure 3. Influence of λ , δ on q_r

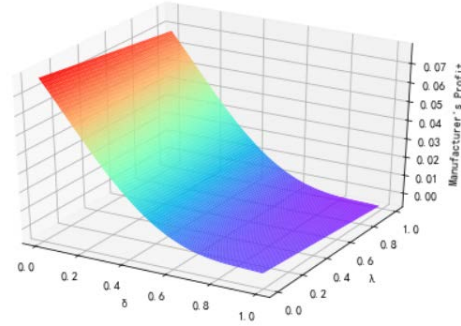


Figure 4. Influence of λ , δ on π_m

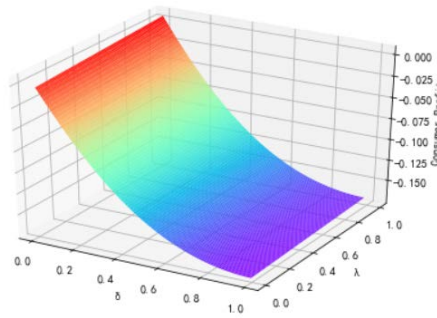


Figure 5. Influence of λ , δ on π_c

Fig. 1 and Fig. 2 show that, as consumers' preference increases, the price of remanufactured products also increases, and the price of new products remains unchanged. The quality coefficient of waste products reported by consumers has less impact on the price of remanufactured products than consumers' preference. At this point, the manufacturer can influence the price of waste products through the preference coefficient of consumers. When consumers' preference increases, the sales volume of new products decreases, and the quality coefficient of waste products reported by consumers has less influence on the sales volume of new products than consumers' preference. In general, improving consumer preference can promote the reuse of remanufactured products, so that manufacturers and consumers can benefit from it and maximize social benefits.

Fig. 3 shows that the quality coefficient of waste products reported by consumers has little influence on the sales of remanufactured products, and the change of consumer preference has a trend of first decreasing, then increasing and then decreasing influence on the sales of remanufactured products. Sales of remanufactured products are maximized when consumer preference is greater. When the consumer preference coefficient is low, manufacturers will reduce the quality of remanufactured products to reduce consumers' demand for remanufactured products. On the contrary, when the consumer preference coefficient is large, manufacturers will improve the quality of remanufactured products to increase consumers' demand for remanufactured products, and manufacturers will also obtain major profits through remanufactured products. In general, manufacturers will adjust the consumer preference coefficient to adapt to different market conditions, so as to maximize their own profits.

As can be seen from Fig. 4 and Fig. 5, with the increase of consumer preference coefficient, both manufacturers and consumers' profits decrease. Therefore, both manufacturers and consumers are willing to reduce the consumer preference coefficient and increase their profits. When the consumer preference coefficient is constant, the quality coefficient reported by consumers increases, the manufacturer's profit decreases, and the consumer's profit increases. The profits of manufacturers and consumers are all against remanufactured products. At this time, government departments should increase subsidies for remanufactured products to make up for the loss of remanufactured products produced and consumed by manufacturers and consumers. When the quality of remanufactured products is determined, the profit of the manufacturer is negatively correlated with the quality of the waste products. Consumers, on the other hand, will lie about the quality of the waste products in order to maximize profits, which will affect the profit of the manufacturer. Therefore, the government should strengthen the supervision of the recycling stage.

4 Conclusion.

In this paper, a closed-loop supply chain model is established by taking into account consumers' different willingness to pay and the uncertainty of recycling quality when the government subsidizes both manufacturers and consumers at the same time. The study found the following conclusions: increasing the consumer preference coefficient can increase the price of remanufactured products; The quality coefficient of waste products reported by consumers has the greatest influence on the profit of manufacturers and consumers. Manufacturers can optimize the profit by adjusting the quality coefficient of waste products reported by consumers. Similarly, consumers make the best profit by reporting the quality factor of waste products. It is found that under certain circumstances, the government subsidy ratio will make the profit of manufacturers and consumers reach the maximum value and minimum value respectively.

References

- [1] Weidong Wang, Xiaoyan Li. Study on coordination strategy of low-carbon supply chain considering competition and information asymmetry[J]. Industrial engineering and management. 09(2019) 1-7.
- [2] Ying Chen, Wenjie Wang, Gengru Meng. Two-channel supply chain coordination of seasonal commodities under asymmetric cost information[J]. Journal of donghua university, 05(2019) 1-10.

- [3] Jian Cao,Xiaoli Yang,Sisi Wu,Gengui Zhou,Ying Xu.Decision of closed-loop supply chain recycling channel considering remanufacturing cost[J].Industrial engineering and management,07(2019) 1-16.
- [4] Savaskan R C,Bhattacharya S, Van Wassenhove L N.Closed-loop Supply Chain Models with Product Remanufacturing[J].Management Science,50(2004) 239-252.
- [5] Zhiqiang Peng,Wenquan Song.Differential pricing and coordination mechanism of closed-loop supply chain under government subsidies[J].Industrial engineering and management,21(2016) 58-66.
- [6] Meng Li.Impact of different government subsidy models on closed-loop supply chain[J]. Economic management,23(2016) 91.
- [7] Wenwei Gong,Hu Li,Rong Zhang.Analysis of competitive closed-loop supply chain recycling and remanufacturing decision model under government subsidies[J].East China economic management,28(2014) 120-125.
- [8] Faxin Cheng,Hanqing Shao,Fangxing Ma.Pricing decision of closed-loop supply chain considering consumers' green preference under differential weighting subsidy[J].Industrial engineering and management, industrial engineering and management, 24(2018) 111-118.
- [9] Tong Shu,Yuqing Xiao,Shu Chen.Study on the impact of government subsidies on the closed-loop remanufacturing supply chain[J].Industrial technology economics,36(2017) 68-73.